On a Recent Proposal of Faster than Light Quantum Communication

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Abstract In a recent paper, A.Y. Shiekh has discussed an experimental set-up which, in his opinion, should make possible faster-than-light communication using the collapse of the quantum wave function. Contrary to the many proposals which have been presented in the past, he does not resort to an entangled state of two systems but he works with a single particle in a superposition of two states—corresponding to its propagation in opposite directions—one of which goes through an appropriate interferometer. The possibility for an observer near the interferometer to introduce or not, at his free will, a phase shifter along one of the paths should allow to change instantaneously the probability of finding the particle in the far-away region corresponding to the other state of the superposition and, correspondingly, to change the intensity of a beam of particles reaching a distant observer. In this paper we show a flaw in the argument: once more, as it has been proved in full generality a long time ago, the process of wave packet reduction cannot be used for superluminal communication.

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1 Introduction

In a recent paper [1], it has been suggested the possibility of resorting to a specific exper-
imental set up to send faster than light signals by taking advantage of the process of wave
packet reduction. The aspect which makes the proposal completely different from all those
which have appeared in the literature up to now derives from the fact that the author does
not resort to an entangled state of two systems one of which is subjected to a measurement,
but he works with a single particle in a superposition of two states, and the measurement
process as well as the ensuing reduction of the wave packet (w.p.r.) involves only one of
the two far-away parts of the wave function. We recall that general proofs that the quantum
process of wave packet reduction does not allow faster-than-light communication have been
presented many years ago [2–6]. However all the just mentioned proofs dealt with situations
involving two entangled states. So, in a sense, the argument of [1] does not fall under such
general proofs and requires a separate comment. In this brief paper we will show that the
process of w.p.r. cannot be used to send superluminal signals to distant observers by fol-
lowing the procedure suggested by Shiekh. Since the author has proposed [7] to resort to an
experimental setup identical to the one considered in [1] in order to enhance the efficiency of
certain quantum information protocols, our analysis proves the inapplicability also of these
proposals.

2 Shiekh's Argument

We briefly review the argument by Shiekh [1]. He considers a particle which is prepared,
at time $t = 0$, in a equal weights superposition[1] of two normalized states, $|h+\rangle$ and $|h-\rangle$,
propagating in two opposite directions, respectively, starting from the common origin of the
x-axis:

$$|\psi, 0\rangle = \frac{1}{\sqrt{2}}[|h+\rangle + |h-\rangle]. \tag{1}$$

Subsequently the state $|h+\rangle$ is injected in an appropriate device behaving in a way similar,
apart from the final stage, to a Mach-Zender interferometer. This device, which splits the
term $|h+\rangle$ of the superposition in two components propagating along two different paths,
is positioned along the positive real x-axis at an appreciable distance from the origin, and
an observer located near it can choose, at his free will, to insert or not a phase-shifter along
one of the two paths. The two wave functions are then recombined by appropriate deflectors
and it is assumed that, by deciding whether or not to insert the phase-shifter, one can pro-
duce a constructive (no phase-shifter in place) or a destructive (the phase-shifter is present)
interference of the two terms in which the impinging state $|h+\rangle$ has been split. Finally, a
detector is placed along the direction of propagation of the final state and it induces wave
packet reduction, since it either detects or fails to detect the particle. We have summarized
the situation for the two considered cases in Figs. 1a and 1b. The pictures simply reproduce
those of [1].

   The author then concludes: *If the sender* (a term used to denote the man who, by his
choice of inserting or not the phase-shifter, can change—according to the author—the am-
plitude of the far away wave function) *arranges for constructive interference, then some*

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[1]The fact that the two state have the same coefficient, is irrelevant; both the argument of the author as well as
our counterargument hold for any superposition of the two states.

Fig. 1 The experimental arrangement considered in [1]. The two cases (**a**) and (**b**) correspond to no phase-shifter inserted or the phase-shifter inserted in one of the arms of the device at right, respectively

3 The Evolution of a Wave Function Crossing the Interferometric Device
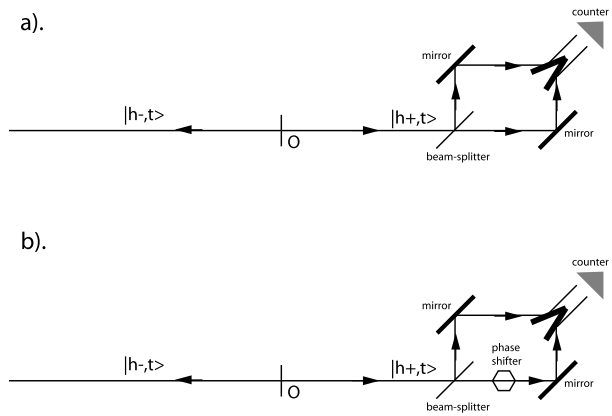
of the particles will be "taken up" by the sender, but none if destructive interference is arranged; in this way the sender can control the intensity of the beam detected by the receiver (the observer located far away where the evolved of $|h-\rangle$ is concentrated). *So, a faster than light transmitter of information (but not energy or matter) might be possible*. We now will reconsider this argument.

In order to make clear the flaw in the argument of [1], let us focus our attention on a spatially well-localized and normalized wave packet $|\phi\rangle$ propagating along the positive direction of the x-axis, with an almost definite momentum, which is then injected in the Mach-Zender like apparatus, as depicted in Fig. 2. For the purpose of our analysis, $|\phi\rangle$ can represent one of the following two situations:

- The term $|h+\rangle$ of an initial superposition of two wave packets, one traveling towards the interferometer, the other traveling in the opposite direction, as in Shiekh's setup;
- A *classical* light impulse, in such a way that $|\langle x|\phi\rangle|^2$ is proportional to its energy density. This possibility makes it clear that, as far as the behavior of $|\phi\rangle$ within the interferometer (and prior to the measurement) is concerned, nothing subtle, highly non-classical goes on.

In the following, for the sake of simplicity, we will refer to $|\phi\rangle$ as representing a quantum particle.

When the wave function $|\phi\rangle$ hits the first beam splitter, half of it keeps moving horizontally and half vertically;[2] the two mirrors A and B simply change the direction of propagation of the two terms of the superposition; finally, when meeting the mirrors C, each of the two terms of the superposition is deflected, describing a particle at an almost definite position propagating at $45°$ with respect to the x-axis towards the detector. In this way these two terms can give rise to an interference pattern.

If we denote by $|\phi_\ell\rangle$ the part of the wave function which followed the lower route in the interferometer, and by $|\phi_u\rangle$ the part which followed the upper route, the effect of the

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[2]Here and in the following, we will ignore the (possible) phase shifts introduced by the beam splitter and/or the mirrors.
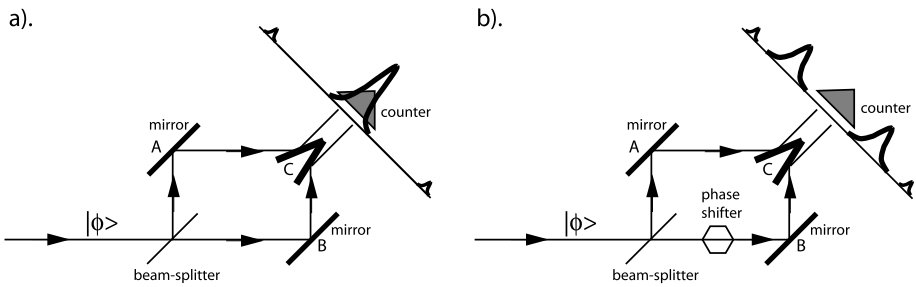
Fig. 2 The actual situation concerning the position probability density distribution when constructive (**a**) and destructive (**b**) interference occurs

interferometer on the initial state $|\phi\rangle$ can be summarized as follows:

$$|\phi\rangle \rightarrow |\phi_f\rangle = \frac{1}{\sqrt{2}}[|\phi_u\rangle + e^{i\varphi}|\phi_\ell\rangle], \tag{2}$$

where φ is the phase introduced by the phase shifter, i.e.

$$\varphi = \begin{cases} 0 & \text{phase shifter not inserted,} \\ \pi & \text{phase shifter inserted.} \end{cases} \tag{3}$$

Note that, for the very structure of the interferometer, the statevectors $|\phi_u\rangle$ and $|\phi_\ell\rangle$ represent two identical, well localized nearby states propagating towards the detector.

Here comes the crucial point: the two mirrors in C cannot be designed in such a way to perfectly superimpose the two terms $|\phi_\ell\rangle$ and $|\phi_u\rangle$ one right on top of the other, in such a way to make them cancel each other when $\varphi = \pi$. The reason is simply that such a device would violate unitarity.[3] A device behaving in this way would be analogous to a glass which is perfectly transparent on one side and perfectly reflecting on the other side: as shown in Fig. 3, it would allow, by an appropriate choice of the two paths and the insertion of a phase shifter on one of them, to get perfectly destructive interference and, consequently, for quantum particles (or classical beams of light) to disappear, which is not possible and goes also against common sense. Accordingly, and as depicted in Figs. 2a and 2b, the state $|\phi_f\rangle$ of (2) represents the superposition of two (essentially identical) wave packets *traveling parallel to each other, at most contiguous*, and such that $\langle\phi_f|\phi_f\rangle = \langle\phi|\phi\rangle = 1$, because of unitarity.[4]

The final state $|\phi_f\rangle$ gives rise to a position density distribution $|\phi_f^\pm(r)|^2 = |\langle r|\phi_f^\pm\rangle|^2$ (r being the running variable along an axis perpendicular to the front of the incoming wave), like those represented in Figs. 2a and 2b. The superscript $+$ refers to the fact that the arrangement is such to induce constructive interference ($\varphi = 0$), while the superscript $-$ refers to

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[3]If we think of $|\phi\rangle$ as representing a classical beam of light propagating in space, then a similar device would violate conservation of energy!

[4]Note that the argument we have developed forbids, precisely for the same reasons, also a partial attenuation of the incoming wave function, i.e. that $\langle\phi_f|\phi_f\rangle < 1$. Any decrease of the probability of the particle being detected in a given interval of the axis orthogonal to the propagation direction of the states emerging from the interferometer must be accompanied by a corresponding increase of finding the particle in the complement of the considered interval.
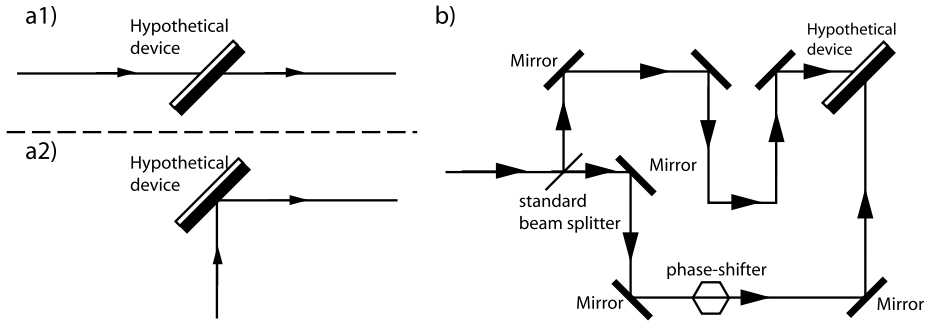
Fig. 3 The hypothetical device underlying the analysis of [1]. At left we have visualized its functioning for an horizontal (**a1**) and a vertical (**a2**) particle, while at right we have illustrated how one can resort to the device to transform a normalized wave function into the neutral element of the Hilbert space, i.e. a function vanishing everywhere, by making the paths of the two terms of the superposition equal in length and by introducing the appropriate phase-shifter in such a way to make them interfere destructively

destructive interference ($\varphi = \pi$). We assume that the detector is small enough to cover only the region right in front of the two mirrors C.

In the case of constructive interference, the counter will fire with probability 1, since the final wave function $\phi_f^+(r)$ is different from zero (practically) only in the interval of the r-axis in which the counter is located, as made explicit in Fig. 2a. In the case in which in the lower arm of the interferometer the phase shifter is introduced, at free will, then the two terms of the superposition will interfere destructively in the (small) region where the detector is placed along the r-axis. In this case the counter will not fire (or better, the probability for it to fire is negligibly small), since the final probability distribution for the position of the particle is the one represented in Fig. 2b, which is associated to the wave function $\phi_f^-(r)$.

As already remarked, the integral of the modulus square of the wave function along the whole r-axis is, in either cases, always equal to 1. Given this fact, can we state, as the author suggests, that the probability of the particle being found in the region at right, due to the destructive interference, is less than 1, and can even be made vanishingly small? Obviously not: the whole process we have described is a unitary evolution of the wave function crossing the interferometer, an evolution which obviously preserves the norm of the state.

To summarize, the situation is the following. When the detector is inserted at the output of the interferometer, with probability 1 it will fire in the case of constructive interference (no phase-shifter inserted) and it will (almost) surely detect nothing when the phase-shifter is inserted. But does this mean that the norm of the initial triggering state has been reduced? Not at all. This simply means that the particle is in the region lying outside the one covered by the counter. Formally, one can say that in this case the reduction has taken place to the linear manifold associated to the projection operator P_{out} on the complement of the interval covered by the counter.

In brief, a reduction process is actually induced by the presence of the detector, but, for the two considered alternatives, the state vector, even before the counter is inserted, is (practically) an eigenstate either of the projection operator P_{in} on the interval covered by the detector (if no phase-shift has been introduced) or of the projection operator P_{out} (if the phase-shifter is present). Obviously, the effect of the reduction is to transform the state vector $|\phi_f\rangle$ either in $P_{in}|\phi_f\rangle$ or in $P_{out}|\phi_f\rangle$, in the two considered cases. Note that, in our case, only one of the norms $\|P_{in}|\phi_f\rangle\|$ and $\|P_{out}|\phi_f\rangle\|$ is different from zero and the it equals 1.

4 The Correct Version of Shiekh's Proposal

It is now an elementary task to make precise the situation devised in [1] and to show that no faster than light communication can be achieved with the mechanism which the author is proposing. Let us take once more the initial state vector considered by the author:

$$|\psi, 0\rangle = \frac{1}{\sqrt{2}}[|h+\rangle + |h-\rangle], \qquad (4)$$

where both $|h+\rangle$ and $|h-\rangle$ are separately normalized. Our analysis has shown that, in the case of constructive interference, the final state will be:

$$|\psi, 0\rangle \rightarrow |\psi_f^+\rangle = \frac{1}{\sqrt{2}}[|\phi_f^+\rangle + |h-\rangle]. \qquad (5)$$

The two terms in the square bracket are both normalized and orthogonal (we neglect the overlapping of the well localized wave functions), and the probability for the counter of the sender to fire is $\langle \psi_f^+ | P_{in} | \psi_f^+ \rangle = 1/2$. Accordingly, the probability for the counter of the receiver to fire is also $1/2$ since for a state like (5), one of the two must fire.

When the phase-shifter is inserted, the analogous of the previous equation is:

$$|\psi, 0\rangle \rightarrow |\psi_f^-\rangle = \frac{1}{\sqrt{2}}[|\phi_f^-\rangle + |h-\rangle]. \qquad (6)$$

In this case, the counter of the sender will not fire for sure, but the global state vector remains the equal weight superposition (6) of two normalized and orthogonal states. This means that the probability for the counter of the receiver to fire is once more equal to $1/2$: the introduction of the phase-shifter has not influenced in any way whatsoever this probability.[5]

5 A Clarifying Example

In this short section we want simply to reconsider the previous argument reducing it to one in which the apparatus at right works precisely as a Mach-Zender interferometer. As well known such an apparatus works in the following way: if things are arranged to yield constructive interference, then the impinging state is transformed into a state propagating with certainty along the horizontal direction, while if things are arranged to yield destructive interference, then the impinging state is transformed into a state propagating with certainty along the vertical direction. Now, the appropriate way to discuss the situation is that of inserting two detectors, one for horizontal and one for vertical final directions of propagation, as we have shown in Fig. 4. Then, if the norm of the impinging state $|\phi\rangle$ equals 1 and constructive interference occurs, the detector along the horizontal path will fire with certainty, while, for a destructive arrangement, it is the one along the vertical path which will fire with certainty.

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[5]Alternatively one might argue as follows: let us replace the counter localizing the particle along r with three adjacent counters, one being the one already considered and the other two covering the whole r axis at left and right of the first counter. Then one of the three counters will detect the particle with probability $1/2$, since the norm of each of the two states $|\psi_f^\pm\rangle$ equals 1. Correspondingly reduction to a state in the region of the counter or in the far-away region at left will take place with equal probabilities.
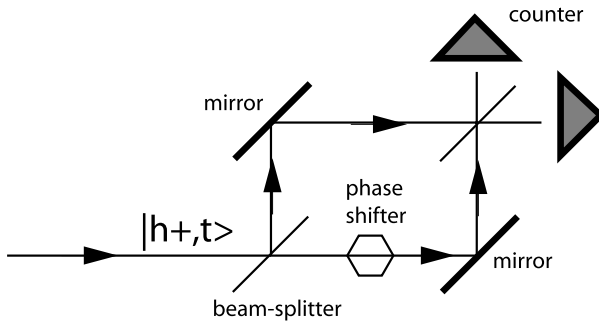
Fig. 4 The situation in the case in which the apparatus at right is a standard Mach-Zender interferometer. In such a case only one of the counters will fire, depending from the presence or absence of the phase-shifter. The figure is more illuminating than the previous ones because the occurrence of constructive and destructive interference corresponds to the triggering of one or the other detector and there is no need to consider the position distribution along the whole r-axis

Taking however into account that the initial state is the one of (1), by exactly the same argument of Sect. 4, we conclude that the final state in this case would be the same as the one of either (5) or (6), with the replacement of the states $|\phi_f^+\rangle$ and $|\phi_f^-\rangle$ by two normalized states describing, respectively, a particle propagating along the horizontal or one propagating along the vertical direction. So, in accordance with the fact that constructive or destructive interference is arranged by the sender, either the detector along the horizontal or the one along the vertical direction might fire, but, in both cases, the probability that it will fire equals 1/2. Correspondingly, the probability of detecting the particle in the situation corresponding to the evolved of the state $|h-\rangle$, remains equal to 1/2 and cannot be changed at free will by the sender.

6 Conclusions

We hope to have made clear that the proposal considered in [1] of resorting to wave packet reduction to send superluminal signals does not work. This is not surprising: the w.p.r. process, in spite of its basically nonlocal features, has been conceived in such a way that it cannot lead to any contradiction with relativistic requirements.

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